Math 220 : Elementary Statistics — Kane Nashimoto

Final Exam Solutions

Spring 2006

1. Inference about proportion.

   (a) $H_0 : \pi = .50$ vs. $H_a : \pi > .50$

   $p = 59/75 = .787$

   $$z^* = \frac{p - \pi_0}{\sqrt{\pi_0(1-\pi_0)/n}} = \frac{.787 - .50}{\sqrt{(.50)(1-.50)/75}} = 4.971$$

   Critical value : $+2.326$ ($z_{.01} = 2.326$)

   Reject H_0. ($\pi > .50$)

   (b) Reject H_0. If a test rejects H_0 at $\alpha = .01$, it will reject H_0 at any $\alpha > .01$.

2. Numerical summary of data.

   Note : $\sum x = 11$; $\sum x^2 = 39$; $n = 6$

   (a) $m = \dfrac{1+2}{2} = 1.500$

   (b) $\bar{x} = \dfrac{\sum x}{n} = \dfrac{11}{6} = 1.833$

   $$s^2 = \frac{\sum x^2 - (\sum x)^2/n}{n-1} = \frac{39 - (11)^2/6}{6-1} = 3.767$$

3. Probability.

   E : {selected student is from out-of-state}

   F : {selected student receives financial aid}

   Note : $P(E) = .75$; $P(F) = .65$; $P(E \cap F) = .55$

   (a) $P(F|E) = \dfrac{P(E \cap F)}{P(E)} = \dfrac{.55}{.75} = .733$

   (b) $P(E \cap F) = .55$ but $P(E)P(F) = (.75)(.65) = .488$

   $\Rightarrow$ Two events are not independent.

4. Normal random variable.

   Note : X, weight of onions, is normal with $\mu = 3.18$ and $\sigma = 0.13$.

   (a) $P(X > 3.00) = P\left(\dfrac{X-\mu}{\sigma} > \dfrac{3.00 - 3.18}{0.13}\right) = P(Z > -1.38)$

   $= 1 - P(Z \le -1.38) = 1 - .0838 = .9162 \Rightarrow 91.6\%$

   (b) $z = -1.28 \because P(Z \le -1.28) \approx .10$

   $z = \dfrac{x-\mu}{\sigma}$; $-1.28 = \dfrac{x - 3.18}{0.13}$; $x = 3.18 + (-1.28)(0.13) = 3.01$

5. Inference about mean with σ unknown.

   (a) $\bar{x} \pm t_{.05/2,\,30-1}\dfrac{s}{\sqrt{n}}$; $9.51 \pm 2.045\dfrac{1.71}{\sqrt{30}}$; 9.51 ± 0.638 ; $(8.872\,,\ 10.148)$

   (b) "10" is contained in the confidence interval as a plausible value of the true mean. Thus, the conjecture is not refuted.

6. Binomial random variable.

   Note: X, number of Hearts, is binomial with $n = 10$ and $\pi = .25$.

   (a) $P(X \leq 2) = .5256$

   (b) $P(3 < X < 7) = P(X \leq 6) - P(X \leq 3) = .9965 - .7759 = .2206$

7. Chi-square goodness-of-fit test.

   $H_0: \pi_1 = .13\,;\ \pi_2 = .87$
   $H_a: \pi_i \neq \pi_{0i}$
   $E_i\ = n\pi_{0i}\,;\ n = 250$
   $E_1 = (250)(.13) = 32.5 \quad E_2 = (250)(.87) = 217.5$

   $$h^* = \sum_i \frac{(O_i - E_i)^2}{E_i} = \frac{(29 - 32.5)^2}{32.5} + \frac{(221 - 217.5)^2}{217.5} = 0.433$$

   Critical value: 2.706 ($h_{.10,\,2-1} = 2.706$)
   Retain H_0.
   (Insufficient evidence to conclude that the counts contradict the manufacturer's claim.)

8. Independent-samples t test by SPSS.

   $H_0: \mu_1 - \mu_2 = 0$ vs. $H_a: \mu_1 - \mu_2 \neq 0$ (1 = "East coast"; 2 = "West coast")
   $t^* = -1.444$ (equal variances assumed)
   two-sided p-value $= .167$ ($> \alpha = .05$)
   Retain H_0.
   Insufficient evidence to conclude that the mean gas prices differ significantly between the two regions.