Math 220: Elementary Statistics — Kane Nashimoto
Exam 3 Solutions
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1. Sampling distribution.

   Note: $\bar{X}$, mean unit price of Porterhouse steak, is normal with $\mu_{\bar{X}} = \mu = 8.45$ and $\sigma_{\bar{X}} = \sigma/\sqrt{n} = 0.63/\sqrt{25}$.

   (a) $P(X > 9.25) = P\left(\dfrac{X-\mu}{\sigma} > \dfrac{9.25-8.45}{0.63}\right) = P(Z > 1.27)$
   $= 1 - P(Z \le 1.27) = 1 - .8980 = .1020$

   (b) $P(\bar{X} > 8.75) = P\left(\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \dfrac{8.75-8.45}{0.63/\sqrt{25}}\right) = P(Z > 2.38)$
   $= 1 - P(Z \le 2.38) = 1 - .9913 = .0087$

2. Inference about proportion.

   (a) $H_0\colon \pi = .75$ vs. $H_a\colon \pi < .75$
   $p = 101/150 = .673$
   $z^* = \dfrac{p-\pi_0}{\sqrt{\pi_0(1-\pi_0)/n}} = \dfrac{.673-.75}{\sqrt{(.75)(1-.75)/150}} = -2.178$
   Critical value: -1.645 ($z_{.05} = 1.645$)
   Reject H_0. ($\pi < .75$)

   (b) p-value $= P(Z \le -2.15) = .0158$

3. Inference about mean with σ known.

   (a) $n = \left(\dfrac{z_{.05/2}\cdot\sigma}{B}\right)^2 = \left(\dfrac{1.960\cdot 2.8}{0.55}\right)^2 = 99.56 \Rightarrow 100$

   (b) $\bar{x} \pm z_{.05/2}\dfrac{\sigma}{\sqrt{n}}$; $67.4 \pm 1.960\dfrac{2.8}{\sqrt{75}}$; 67.4 ± 0.634 ; $(66.766\,,\ 68.034)$

4. Inference about mean with σ unknown.

   (a) $H_0\colon \mu = 1.28$ vs. $H_a\colon \mu \neq 1.28$
   $t^* = \dfrac{\bar{x}-\mu_0}{s/\sqrt{n}} = \dfrac{1.36-1.28}{0.87/\sqrt{20}} = 0.411$
   Critical values: ± 2.093 ($t_{.05/2,\,20-1} = 2.093$)
   Retain H_0. ($\mu \approx 1.28$)

   (b) The 95% confidence interval should contain $\mu_0 = 1.28$. The result of (a) indicates that 1.28 is a plausible value of μ and, therefore, must be contained in the confidence interval.

   (c) $\bar{x} \pm t_{.05/2,\,20-1}\dfrac{s}{\sqrt{n}}$; $1.36 \pm 2.093\dfrac{0.87}{\sqrt{20}}$; 1.36 ± 0.407 ; $(0.953\,,\ 1.767)$

   (Note that this interval does contain 1.28.)