Math 220 : Elementary Statistics — Kane Nashimoto

Final Exam Solutions

Fall 2005

1. Probability.

   (a) $P(\text{woman} \mid \text{woman}) = \dfrac{12}{19} = .632$

   (b) $P(\text{all men}) = \dfrac{7}{20} \cdot \dfrac{6}{19} \cdot \dfrac{5}{18} = .031$

2. Inference about proportion.

   (a) Let $\pi = .5$.
   $n = \pi(1-\pi)\left(\dfrac{z_{.10/2}}{B}\right)^2 = (.5)(1-.5)\left(\dfrac{1.645}{.075}\right)^2 = 120.27 \ \Rightarrow \ 121$

   (b) $p \pm z_{.05/2}\sqrt{\dfrac{p(1-p)}{n}}$; $.205 \pm 1.960\sqrt{\dfrac{(.205)(1-.205)}{200}}$; $.205 \pm .056$; $(.149\,,\ .261)$

3. Binomial random variable.

   (a) X, number of flowers with smooth leaves, is binomial with $n = 15$ and $\pi = .40$.
   $P(X = 5) = P(X \le 5) - P(X \le 4) = .4032 - .2173 = .1859$

   (b) Y, number of flowers with hairy leaves, is binomial with $n = 15$ and $\pi = .60$.
   $P(Y > 8) = 1 - P(X \le 8) = 1 - .3902 = .6098$

4. Graphical summary of data.
   Note : $\sum x = 108$; $\sum x^2 = 2222$; $n = 7$

   (a) $s^2 = \dfrac{\sum x^2 - (\sum x)^2/n}{n-1} = \dfrac{2222 - (108)^2/7}{7-1} = 92.619$

   (b) The distribution of the data is positively skewed.

5. Experimental design.

   (a) Assignment of the participants to the treatment groups was not randomized.

   (b) An individual, who is unaware of which group receives what treatment, must give the treatments and measure the levels of depression.

6. Sampling distribution.

   (a) $\bar{X}$, mean number of paperclips, is approximately normal with $\mu_{\bar{X}} = \mu = 103.7$ and $\sigma_{\bar{X}} = \sigma/\sqrt{n} = 1.4/\sqrt{50}$. This is due to the central limit theorem.

   (b) $P(\bar{X} < 103.5) = P\left(\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} < \dfrac{103.5 - 103.7}{1.4/\sqrt{50}}\right) = P(Z < -1.01) = .1562$

7. Chi-square test of independence.

   (a) H_0: Gender and political affiliation are independent.
   H_a: Gender and political affiliation are not independent.

   Observed and marginal counts:

39	18	57
87	24	111
126	42	168

   $E_{ij} = (O_{i\cdot})(O_{\cdot j})/n$
   $E_{11} = (\ 57\ )(126)/168 = 42.75 \quad E_{12} = (\ 57\ )(42)/168 = 14.25$
   $E_{21} = (111)(126)/168 = 83.25 \quad E_{22} = (111)(42)/168 = 27.75$

   $$h^* = \sum_i \sum_j \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$
   $$= \frac{(39 - 42.75)^2}{42.75} + \frac{(18 - 14.25)^2}{14.25} + \frac{(87 - 83.25)^2}{83.25} + \frac{(24 - 27.75)^2}{27.75} = 1.991$$

   Critical value: 6.635 $\ (h_{.01,\,(2-1)(2-1)} = 6.635)$
   Retain H_0.
   (Insufficient evidence to conclude that one's political affiliation depends on gender.)

   (b) $E_{ij} > 5$ for all i, j
   $\Rightarrow$ Assumption is satisfied.

8. Paired-samples t test by SPSS.

   $H_0: \mu_\delta = 0$ vs. $H_a: \mu_\delta < 0$ ("Beginning" minus "Eight weeks")
   $t^* = -2.315$
   one-sided p-value $= .041 \div 2 = .0205$ $(< \alpha = .05)$
   Reject H_0.
   The mean level of interest was significantly higher eight weeks into the semester than it was at the beginning of the semester.