You must turn in solutions to all the problems. The problem with your name is the one you will present in class.

Let V be an n dimensional vector space over a field $\mathbb{F}$. Let T be a linear transformation from V to itself.

1. [**10pts – Dana**] Suppose that $p(x)$, the characteristic polynomial for T, is irreducible. Prove that there exists a basis β for V such that $[T]_\beta$ is the companion matrix for $p(x)$.

2. [**10pts – Emily**] Let $p(x)$ and $q(x)$ be monic irreducible polynomials over $\mathbb{F}$. Put $h(x) = p(x)q(x)$, and let $C_{p(x)}, C_{q(x)}$, and $C_{h(x)}$ be the companion matrices of the respective polynomials. Is it necessarily true that $C_{h(x)}$ is similar to the block diagonal sum of $C_{p(x)}$ and $C_{q(x)}$? If so, then prove it. Otherwise, construct a counter example.

3. [**10pts – Amanda**] Prove that any linear transformation on a finite dimensional vector space may be placed in Rational Canonical Form. (Use the methods we developed in class, don't just copy something from a book!)

4. [**10pts – Kristin**] Prove the Cayley-Hamilton Theorem. (Use the methods we developed in class, don't just copy something from a book!)

5. [**10pts – Greg**] Let K/F be an extension of fields of finite degree n. Show that for any $\alpha \in K$, multiplication by α induces a linear transformation of K viewed as a vector space over F.

6. [**10pts – Adam**] Let K/F be an extension of fields of finite degree n. Prove that K is isomorphic to a subfield of the ring of $n \times n$ matrices over F.