

Math 205 Practice Fi- Name (print neatly): _____
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THIS IS A PRACTICE EXAM. IT IS INTENTIONALLY MUCH HARDER THAN THE ACTUAL FINAL WILL BE.

Honor Pledge: I understand that it is a violation of the JMU honor code to give or receive unauthorized aid on this exam. Furthermore, I understand that I am obligated to report any violation of the honor code by other students that I may become aware of, and that my failure to do so is itself a violation.

(Note: This is a PRACTICE exam. You can get help from whomever you want. This is only on here so that it will look like the real final.)

Signature: _____

OFFICIAL
USE ONLY

1. (20)_____

2. (20)_____

3. (20)_____

4. (8)_____

5. (8)_____

6. (6)_____

7. (6)_____

8. (12)_____

9. (50)_____

Total: _____

Curved:_____

Exam Score

INSTRUCTIONS — READ THIS NOW

• **Don't Panic!** This test will be curved to a mean of 80% with a standard deviation of 12%. So, do your best, and don't freak out if there are problems you can't do.

• This test has 9 problems on 15 pages worth a total of 150 points. Look over your test package **right now**.

• Neatly print your name and sign the honor pledge **right now**.

• If you finish in the last ten minutes of the testing period, please be courteous to those still working by remaining quietly seated.

• **You do not need to simplify your answers.**

1. [20pts (4pts each)] Compute the following derivatives

(a) $\frac{d}{dx} \frac{e^{x^2}}{x}$

(b) $\frac{d}{dx} \int_1^{\sqrt{x}} e^{t^2} dt$

(c) $\frac{d}{dx} (x^2 + 1)^{x^3+1}$

$$(d) \frac{d}{dx} \sqrt{1 + \sqrt{2 + x}}$$

$$(e) \frac{d}{dx} \left(\frac{3^{2x} \sqrt{x^3 - 1}}{\sqrt[5]{x - 1}} \right)$$

2. [20pts (4pts each)] Compute the following limits

(a) $\lim_{x \rightarrow 0} \frac{x^2 + 1}{x}$

(b) $\lim_{x \rightarrow \infty} e^{-x}(x^{20} + 6)$

(c) $\lim_{x \rightarrow \infty} (1 - 1/x)^x$

(d) $\lim_{x \rightarrow 0^+} x \ln(x)$

(e) $\lim_{x \rightarrow 0} \left(\frac{x}{1 - e^x} \right)$

3. [20pts (4pts each)] Compute the following integrals

(a) $\int 3x^2 + 5x - 1 \, dx$

(b) $\int (x^2 + 1)^{99} x \, dx$

(c) $\int 6x^5 + 4x^3 + 2x^{-1} \, dx$

(d) $\int_0^1 x^2 e^{x^3} dx$

(e) $\int x^3(x^4 + 3) dx$

4. [8pts] Bob the biologist is modeling yeast growth in beer. In the beginning, he adds a small amount of yeast to a mixture of water, barley, and hops. Initially, the yeast grow at an exponential rate. However, as the yeast grow, they produce alcohol which poisons their environment. Eventually the yeast reach a maximum population which is sustained for a little while. Then the alcohol build up in their environment becomes so great that it kills off all of the yeast and the population goes to zero.

Construct a function that could reasonably model yeast growth in beer and explain how each feature of your function is involved in the model. You may just use labeled constants (e.g a, b, c, d, \dots) instead of trying to figure out reasonable numbers. Be sure to explain what your constants means. Your function can even have derivatives and integrals in it if that makes it easier to describe. It will probably help to draw a well-labeled graph of your function, first.

5. [8pts] Compute the following integral:

$$\int \frac{x}{\sqrt{x^2 - 1}} dx.$$

6. **[6pts]** In your own words, explain what two concepts are related by the fundamental theorem of calculus.

7. **[6pts]** Now give a graphical proof of the fundamental theorem of calculus, and explain how your proof illustrates the concepts you mentioned in the previous problem.

8. [12pts] More word problems! Fun!!!

- (a) Ryan is pointing his telescope at a rising weather balloon. Ryan is 1 mile away from the balloon launch site. The balloon's height (in miles) is given as a function of time (in hours) by $h(t) = 12(1 - 2^{-t})$. What is the rate of change of the angle, θ , that Ryan's sombrero makes with the ground two hours after the launch (i.e. when $t = 2$)?
- (b) The Zyberwatt corporation is manufacturing COWbangos. The marketing department has determined that if COWbangos are priced at p dollars each, then Zyberwatt can expect to sell approximately $f(p) = 1000000e^{-0.25p^2}$ of them. What price should Zyberwatt set to maximize the total sales revenue of COWbangos? (Total sales revenue is just the number of units sold times the price.)

9. [50pts (2pts each)] **True or False: With Justification.** For each of the following problems, decide if the statement is true or false and explain your reasoning.

(a) **True** **False** $\frac{d}{dx}e^x = e^x.$

(b) **True** **False** $\sqrt{5} = \sqrt{2} + \sqrt{3}.$

(c) **True** **False** $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}.$

(d) **True** **False** $\frac{d}{dx}x^e = ex^{e-1}.$

(e) **True** **False** If $x > 0$, then $\int_e^x \frac{1}{t} dt = \ln(x) - 1.$

(f) **True** **False** $\int_{-1}^1 x^3 dx = 0$

(g) **True** **False** Suppose that the function, f , is defined as

$$f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$

then $\int_0^x f(t) dt = |x|.$

(h) **True** **False** Suppose that $f(x) \leq 3$ and $F(x)$ is an antiderivative of f , then $F(2) - F(0) \leq 3.$

- (i) **True** **False** Suppose that f and g are continuous functions, then
$$\int_0^2 f(x) + g(x) dx = \int_0^1 f(x) dx + \int_1^2 g(x) dx.$$
- (j) **True** **False**
$$\lim_{n \rightarrow \infty} \sum_{k=1}^n (3(0 + k(2/n))) \frac{2}{n} = 6.$$
- (k) **True** **False** If $F(x)$ is an anti-derivative of $f(x)$, then $f'(x) = F(x)$.
- (l) **True** **False**
$$\lim_{n \rightarrow \infty} \left(\frac{n+2}{n} \right)^n = e^2.$$
- (m) **True** **False**
$$\lim_{x \rightarrow 0} \frac{3x^3 + 2x + 1}{x^3 + 2} = 3.$$
- (n) **True** **False**
$$\lim_{x \rightarrow \infty} \frac{5x^2 + 6x - 1}{x^2 + 2} = 5.$$
- (o) **True** **False** The fundamental theorem of calculus implies that any antiderivative of the derivative of a continuous function is equal to the original function.

(p) **True** **False** The fundamental theorem of calculus implies that the derivative of any antiderivative of a continuous function is equal to the original function.

(q) **True** **False** The fundamental theorem of calculus implies that for any continuous function, f , the area accumulation function of f , $A(x) = \int_a^x f(t) dt$, is an antiderivative of f .

(r) **True** **False** $\int_0^1 x^2 dx = \int_0^1 \clubsuit^2 d\clubsuit$

(s) **True** **False** If you hiked the Appalachian Trail through Virginia, then at some point along your trip your altitude is exactly equal to your average altitude over the entire trip.

(t) **True** **False** $\int \frac{1}{\text{cabin}} d\text{cabin} = \ln(\text{cabin}) + C.$