

Math 248 Computers and Numerical Algorithms–Pruett

EXTRA GRADED HW ASSIGNMENT 3

Assigned: Monday, 13 October, 2008.

Due: Monday, 20 October, 2008.

Notes: Please use only a hand calculator for this assignment (i.e., no Fortran programs). You may work with other students, but each student is to turn in an individual assignment.

Consider the function $f(x) = x - 2\sin(x)$.

1. Show that $f(x)$ has at least one root in the interval $[\pi/2, \pi]$ (Hint: use the existence theorem).
2. Now show that $f(x)$ has *only* one root in the interval above. (Hint: use the uniqueness theorem).
3. Carefully sketch the functions $y = x$ and $y = 2\sin(x)$ on the same axes. Estimate the point of intersection, which is the root of $f(x)$.
4. Find an iteration function $g(x)$ whose fixed point $p = g(p)$ is the root of $f(x)$.
5. Find a subinterval $[\pi/2, ?]$ for which your fixed-point iteration $x_{i+1} \leftarrow g(x_i)$ is guaranteed to converge. (Hint: use the FPI theorem.)
6. Compute the first 20 iterates using a reasonable initial guess. Is the iteration converging? At about the rate you expected?
7. Find the root also by the secant method. Compare the number of iterations with that of FPI.
8. Find the root also by Newton's method. Compare the number of iterations with that of FPI and the secant method.

PS: For all of the above, make sure your calculator is set to RADIANS.