

**Exam II**  
**Math 237, Spring 2008**

Answer each of the following problems as completely as you can. You must show your work to receive credit. Progress in the right direction will be worth partial credit.

Do not use a calculator.

DRAW A BOX AROUND YOUR FINAL ANSWER TO EACH PROBLEM.  
MAKE THAT ANSWER LEGIBLE.

- (1) (a) For  $f : \mathbb{R}^n \rightarrow \mathbb{R}$ , what is the definition of  $\nabla f$ ? What does this represent?

- (b) What are three significant facts about  $\nabla f$ ?

- (c) Give an example of a function of two variables such that

$$\lim_{(x,y) \rightarrow (1,1) \text{ along the line } y=x} f(x,y) = 3, \quad \text{and}$$
$$\lim_{(x,y) \rightarrow (1,1) \text{ along the line } x=1} f(x,y) \quad \text{dne}$$

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(2) (a)

$$f(x, y, z, w) = e^{e+z} - y^3 w^2 + \cos(4zw)$$

Compute  $\nabla f$ .

(b) What is the equation of the plane tangent to the surface  $z = x + 4 \cos y$  at the point  $(1, 0, 5)$ ?

(3) Find and classify the critical points of

$$f(x, y) = \frac{x^3}{3} - \frac{x^2}{2} + \frac{y^3}{3} - 4y$$

- (4) The temperature at a point  $(x,y)$  on a metal plate is given by

$$T(x, y) = \frac{xy}{1 + x^2 + y^2}$$

degrees Celsius.

- (a) Find the rate of change of  $T$  at  $(1, 1)$  in the direction of the point  $(3, 0)$

- (b) A spider on this plate wants to travel in the direction of fastest decrease in temperature. Find a unit vector in the desired direction.

- (5) An open cylindrical can has an inside radius of 2 cm and an inside height of 5 cm. Use differentials to approximate the volume of metal in the can if it is 0.01 cm thick.