

**Constrained Extrema**  
**Math 237, Spring 2008**

Use the method of Lagrange multipliers to solve each of the following:

- (1) Minimize  $x^2 + y^2$  on the hyperbola  $xy = 1$
  
- (2) Show that the square has largest area of all rectangles with a given perimeter.
  
- (3) Minimize  $x^4 + y^4 + z^4$  on the plane  $x + y + z = 1$
  
- (4) Maximize  $f(x, y, z) = 3x - 2y + z$  on the sphere  $x^2 + y^2 + z^2 = 14$
  
- (5) Number 16 in 16.10
  
- (6) Number 14 in 16.10
  
- (7) Show that of all the triangles inscribed in a fixed circle, the equilateral triangle has the largest: (a) product of the lengths of the sides; (b) sum of squares of the lengths of the sides.
  
- (8) Maximize  $x^2 + y^2$  on the curve  $x^4 + 7x^2y^2 + y^4 = 1$ .
  
- (9) A plane passes through the point  $(a, b, c)$ . Find its intercepts with the coordinate axes if the volume of the solid bounded by the plane is to be at a minimum.