

PRACTICE MIDTERM III
Math 236, Fall 2007

NOTE: this practice exam is intended to give students an idea of the formt, type of questions, and range of topics that may appear on the upcoming midterm. It is NOT exhaustive in either respect. Doing the practice exam is not a wise substitute for comprehensive review.

Answer each of the following problems as completely as you can. You must show your work to receive credit. Calculators are not allowed.

(1) (20 *points*)

(a) give an exact statement of what it “ $\{a_n\}$ does not converge” *means*.
Hint for practice exam: epsilon will appear in this statement

(b) Give, if possible, an example of a sequence that is bounded but not convergent.

(c) Is it true that every monotone sequence conerges? Why or why not?

(d) What is

(i) the Archimedean Property of \mathbb{R} ?

(ii) the Least Upper Bound property of \mathbb{R} (also called the Axiom of Completeness)?

(2) (20 *points*) Evaluate each of the following. If the requested quantity does not exist, show or explain why it does not:

(a)

$$\lim_{x \rightarrow 1} \frac{1 - x}{\sin(x - 1)}$$

(b)

$$\lim_{x \rightarrow 0} x\{x^{-3} + 2x + \cos x\}$$

(c)

$$\lim_{x \rightarrow 3} \frac{0}{3 - x}$$

(3) (20 *points*) Prove directly from the definition of sequence convergence that

$$\lim_{n \rightarrow \infty} \left\{ 3 + \frac{1}{n} \right\}^2 = 9$$

4

(4) (20 *points*) Exercise 31 of 11.1.

(5) (20 *points*) State and prove the Monotone Convergence Theorem.