

PRACTICE MIDTERM I
Math 236, Fall 2007

NOTE: this is a practice exam. It is intended to give students an idea of the format, type of questions, and range of topics that may appear on the upcoming midterm. It is NOT exhaustive in either respect. Doing the practice exam is not a wise substitute for comprehensive review.

Also, the questions on this exam have not been vetted as carefully as questions on a real exam will be.

Answer each of the following problems as completely as you can. You must show your work to receive credit. Calculators are not allowed.

- (1) (20 *points*)
 - (a) Draw the graph of an invertible function. Explain why the function you drew is invertible.
 - (b) On the same axes as part (a) above, sketch the graph of the inverse of your original function.

- (c) What is the definition of a^r , where $a > 0$ and r is irrational?

- (d) What is the *definition* of the function $\ln x$, as developed in the Exercises?

- (e) Briefly explain the use of μ -multipliers to solve linear differential equations.

(2) (20 points) Evaluate each of the following. If the requested quantity does not exist, show or explain why it does not:

(a)

$$\left(\sqrt{1 + \ln^2 x}\right)'$$

(b)

$$\int \frac{\cosh(\sqrt{x})}{\sqrt{x}} dx$$

(c)

$$\int_0^e \frac{1}{x + e} dx$$

(d)

$$\int_0^1 2^5 x dx$$

(3) (20 *points*) Find $y(t)$, given that

$$\frac{dy}{dt} + \frac{1}{t^2} y = e^t$$

(4) (20 *points*) Sixty percent of a radioactive substance decays in the period from 2000 to 2010.

(a) Find a formula that will express the amount of substance present t years after 2000.

(b) What is the half-life of this substance?

- (5) (20 *points*) A rectangle has one side along the x - axis, one corner at the origin, and another vertex on the curve $y = e^{-x}$, where $x > 0$. What is the maximum area the rectangle can have?