name: ______________ e-mail: ____________________

check if faculty: ______ class and professor: _________________

The problems of the week are available online at
http://www.math.jmu.edu/~rosenhjd/POTW.html.

Problem of the Week Nine

Our tour of the history of probability theory has brought us to 1948. That was the year Claude Shannon published his two-part paper "A Mathematical Theory of Communication," thereby giving birth to the branch of mathematics known seductively as "Information Theory." To this day it remains a central subject in applied probability.

Shannon's framework involved a receiver who knows he will receive one of a finite number of messages, with each message having some known probability of being sent. He then defined the quantity of information in a message as the reduction in uncertainty that resulted when an actual message was subsequently received. There is a precise mathematical formula for measuring this reduction in uncertainty, but that need not detain us here.

If you have ever played the game of higher and lower you are already aware of some of the main points of Shannon's system. I am thinking of a number x between one and one thousand, and you are to determine x with as few guesses as possible. Each time you call out a number I reply with "Higher!" or "Lower!" until you get it right. The best strategy, of course, is to begin with 500, for in this case you are certain that half the numbers will be eliminated from consideration by my answer. In other words, 500 represents the point of maximum uncertainty regarding the value of x; it is, with equal probability, greater than 500 or smaller than 500. Consequently, it is the value that forces me to give you the maximum amount of information. Had you instead started with, say, 10, then there is a very small probability you would hit the jackpot by hearing me say, "Lower!" But it is far more likely that I would say "Higher!" and that presents you with very little information indeed. Messages heard with high probability convey less information than those with small probability.

In honor of Shannon's seminal contributions to this field, how about a problem of the week that involves some irrelevant information (yes, that's a hint!):

A machine contains 1000 **red marbles,** 1000 **green marbles and** 1000 **blue marbles. While your back is turned, the machine churns out a random sample containing an unknown number of marbles and places them in an opaque bag. You reach into the bag without looking and remove one marble at random. What is the probability that this marble is red?**

Solutions are due **Monday, April 21** by 5:00 to Jason Rosenhouse in Roop 121. One weekly winner will receive a five dollar gift card to Greenberry's, and will be chosen randomly from among the correct answers. As always, please give a line or two of explanation to accompany your answer.