

Solution to Problem of the Week Six

In America we write “HUMOR.” In England the same word is spelled, “HUMOUR.” Suppose you are at a party where two thirds of the people are American and one third are British. One person is chosen at random to write the word on a piece of paper. A letter is then chosen randomly from the word that was written and is seen to be a “U.” What is the probability that the person who wrote the word is British?

The probability is $\frac{5}{11}$.

Define the following events:

B = A British person wrote the word.

U = The randomly selected letter is a “U”.

Our goal is to find $P(B|U)$. According to Bayes’ Theorem we have

$$P(B|U) = \frac{P(B)P(U|B)}{P(U)}.$$

We are given that $P(B) = \frac{1}{3}$. Since the British spelling of “HUMOUR” has two U ’s out of six letters, we find that $P(U|B) = \frac{1}{3}$ as well.

To evaluate $P(U)$, let us first define the event A to be “An American wrote the word.” Then by the law of total probability we have

$$\begin{aligned} P(U) &= P(A)P(U|A) + P(B)P(U|B) \\ &= \left(\frac{2}{3}\right) \left(\frac{1}{5}\right) + \left(\frac{1}{3}\right) \left(\frac{1}{3}\right) = \frac{11}{45}. \end{aligned}$$

Plugging everything into Bayes’ theorem now gives:

$$P(B|U) = \frac{\frac{1}{3} \left(\frac{1}{3}\right)}{\frac{11}{45}} = \frac{5}{11}.$$