name: _____________ e-mail: ____________________

check if faculty: _____ class and professor: ________________

The problems of the week are available online at
http://www.math.jmu.edu/∼rosenhjd/POTW.html.

Problem of the Week Six

We come, now, to Thomas Bayes, mathematician and Presbyterian minister. He took up the following problem: Let us suppose an experiment is carried out numerous times. We find that the event E occurs in x runs of the experiment and fails to occur in y runs. What can be said regarding the probability of E on a single trial? Problems of this sort are sometimes said to be examples of "inverse probability." Prior to Bayes problems in probability tended to proceed from causes to effects. We flip a fair coin multiple times, and try to say something intelligent about the sequence of heads and tails we are likely to observe. Inverse probability reverses this thinking. We have a sequence of H's and T's, and ask how likely it is that the coin that produced them was fair.

Bayes occupies a curious position in the history of mathematics. His name is nowadays attached to a major school of philosophical thought regarding the proper interpretation of probability. That notwithstanding, many historians of mathematics seem unable to dismiss him quickly enough. Florence David, in her book *Games, Gods and Gambling*, does not mention him at all. Ian Hacking, in *The Emergence of Probability*, mentions him only in passing. Isaac Todhunter, in *A History of the Theory of Probability* devotes a chapter to him, but at six pages it is the shortest chapter in the book. It also includes statements like this:

> Bayes begins, as we have said, with a brief demonstration of the general laws of the Theory of Probability; this part of his

> essay is excessively obscure, and contrasts most unfavorably with the treatment of the same subject by De Moivre.
>
> Bayes gives the principle by which we must calculate the probability of a compound event.
>
> Suppose we denote the probability of the compound event by $\frac{P}{N}$, the probability of the first event by z, and the probability of the second on the supposition of the happening of the first by $\frac{b}{N}$. Then our principle gives us $\frac{P}{N} = z \times \frac{b}{N}$, and therefore $z = \frac{P}{b}$. This result Bayes seems to present as something new and remarkable; he arrives at it by a strange process, and enunciates it as his Proposition Five in these obscure terms:
>
> "If there be two subsequent events, the probability of the 2nd $\frac{b}{N}$ and the probability of both together $\frac{P}{N}$, and it being first discovered that the second event has happened, from hence I guess that the first event has also happened, the probability I am in the right is $\frac{P}{b}$."
>
> Price himself gives a note which shews a clearer appreciation of the proposition than Bayes had.

Ouch! Price, incidentally, is the fellow who arranged for the posthumous publication of Bayes' essay.

Well, Bayes' may not have been the clearest writer in the world, but that doesn't mean we can't show our appreciation of his work with this week's problem:

In America we write "HUMOR." In England the same word is spelled, "HUMOUR." Suppose you are at a party where two thirds of the people are American and one third are British. One person is chosen at random to write the word on a piece of paper. A letter is then chosen randomly from the word that was written and is seen to be a "U." What is the probability that the person who wrote the word is British?

Solutions are due **Friday, March 21** by 5:00 to Jason Rosenhouse in Roop 121. One weekly winner will receive a five dollar gift card to Greenberry's, and will be chosen randomly from among the correct answers. As always, please give a line or two of explanation to accompany your answer.