

The problems of the week are available online at
<http://www.math.jmu.edu/~rosenhjd/potw.html>.

Problem of the Week Three

Imagine that we have forty cards consisting of the Ace through Ten of each suit. These cards are thoroughly shuffled and four cards are then selected at random. What is the probability of selecting one card from each suit?

There are many ways to approach this problem, but here is the method that seems most straightforward to me. We imagine selecting one card at a time, and seek the probability that all four cards are of different suits. The first card can be anything. After this selection, there are 39 cards remaining, 30 of which are of a different suit from the first card. This gives a probability of $\frac{30}{39}$ that the second card will be of a different suit from the first card. Likewise, there is a probability of $\frac{20}{38}$ that the third card will be of a different suit from either of the first two, and a probability of $\frac{10}{37}$ that the final card will be of a different suit from the first three. It follows that the probability that all four cards are of different suits is

$$\frac{30}{39} \times \frac{20}{38} \times \frac{10}{37} = \frac{6,000}{54,834} = \frac{1000}{9139}.$$