Solution to Problem of the Week One

How many times must you roll a standard six-sided die before the probability of rolling a six on at least one roll is greater than one half?

Four rolls are necessary. Note that the probability of obtaining a six on any given roll is $\frac{1}{6}$. It follows that the probability of not rolling a six is $\frac{5}{6}$. Consequently, the probability of not rolling a six on any of n consecutive rolls is $\left(\frac{5}{6}\right)^n$. The solution to the problem will then be the smallest value of n for which the inequality

$$1 - \left(\frac{5}{6}\right)^n \geq \frac{1}{2}$$

is satisfied. A little trial and error now shows that $n = 4$ is the number we seek.

The probability of rolling a six on at least one of three rolls of a die is $\frac{91}{216} \approx .4213$. The probability of rolling a six on at least one of four rolls of a die is $\frac{671}{1296} \approx .5177$.