Math 220 : Elementary Statistics — Kane Nashimoto

Exam 3 Solutions

Spring 2007

1. Inference about mean with σ unknown.

   (a) $H_0 \colon \mu = 1.10$ vs. $H_a \colon \mu \neq 1.10$

   $t^* = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{1.16 - 1.10}{0.17/\sqrt{20}} = 1.578$

   Critical values : ± 2.861 $(t_{.01/2,\,20-1} = 2.861)$

   Retain H_0. $(\mu \approx 1.10)$

   (b) $\bar{x} \pm t_{.01/2,\,20-1} \dfrac{s}{\sqrt{n}}$; $1.16 \pm 2.861 \dfrac{0.17}{\sqrt{20}}$; 1.16 ± 0.109 ; $(1.051\,,\ 1.269)$

   (c) The 99% confidence interval should contain $\mu_0 = 1.10$. The result of (a) indicates that 1.10 is a plausible value of μ and, therefore, must be contained in the confidence interval.

2. Inference about proportion.

   (a) $p = 273/390 = .700$

   $p \pm z_{.05/2} \sqrt{\dfrac{p(1-p)}{n}}$; $.700 \pm 1.960 \sqrt{\dfrac{(.700)(1-.700)}{390}}$; $.700 \pm .045$; $(.655\,,\ .745)$

   (b) Let $\pi = p$.

   $n = \pi(1-\pi)\left(\dfrac{z_{.05/2}}{B}\right)^2 = (.700)(1-.700)\left(\dfrac{1.960}{.04}\right)^2 = 504.21 \ \Rightarrow\ 505$

3. Sampling distribution.

   (a) $\mu_{\bar{X}} = \mu = 61.0$

   (b) $\sigma_{\bar{X}} = \sigma/\sqrt{n} = 4.0/\sqrt{20}$

   (c) $P(\bar{X} > 60.0) = P\left(\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} > \dfrac{60.0 - 61.0}{4.0/\sqrt{20}}\right) = P(Z > -1.12)$

   $= 1 - P(Z \leq -1.12) = 1 - .1314 = .8686$

4. Inference about mean with σ known.

   (a) $H_0 \colon \mu = 33.0$ vs. $H_a \colon \mu > 33.0$

   $z^* = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{35.0 - 33.0}{4.0/\sqrt{30}} = 2.739$

   Critical value : $+1.645$ $(z_{.05} = 1.645)$

   Reject H_0. $(\mu > 33.0)$

   (b) p-value $= P(Z \geq 2.75) = 1 - P(Z < 2.75) = 1 - .9970 = .0030$